

REVIEW SET 4A

- 1 Find the following *without* using a calculator. Show all working.
- a** $\log_4 64$ **b** $\log_2 256$ **c** $\log_2(0.25)$ **d** $\log_{25} 5$ **e** $\log_8 1$
f $\log_6 6$ **g** $\log_{81} 3$ **h** $\log_9(0.\bar{1})$ **i** $\log_{27} 3$ **j** $\log_k \sqrt{k}$
- 2 Without using a calculator, find: **a** $\log \sqrt{10}$ **b** $\log \frac{1}{\sqrt[3]{10}}$ **c** $\log 10^a \times 10^{b+1}$
- 3 Find x if: **a** $\log_2 x = -3$ **b** $\log_5 x \doteq 2.743$ **c** $\log_3 x \doteq -3.145$
- 4 Write as logarithmic equations: **a** $P = 3 \times b^x$ **b** $m = \frac{n^3}{p^2}$
- 5 Write the following equations without logarithms:
a $\log_2 k \doteq 1.699 + x$ **b** $\log_a Q = 3 \log_a P + \log_a R$ **c** $\log A \doteq 5 \log B - 2.602$
- 6 Solve for x , giving your answer correct to 4 significant figures:
a $5^x = 7$ **b** $20 \times 2^{2x+1} = 500$
- 7 The weight of radioactive substance after t years is $W_t = 2500 \times 3^{-\frac{t}{3000}}$ grams.
a Find the initial weight.
b Find the time taken for the weight to reduce to 30% of its original value.
c Find the percentage weight loss after 1500 years.
d Sketch the graph of W_t against t .
- 8 Solve for x : $16^x - 5 \times 8^x = 0$

REVIEW SET 4B

- 1 Without using a calculator, find the base 10 logarithms of: **a** $\sqrt{1000}$ **b** $\frac{10}{\sqrt[3]{10}}$ **c** $\frac{10^a}{10^{-b}}$
- 2 Solve for x :
a $\log x = 3$ **b** $\log_3(x+2) = 1.732$ **c** $\log_2\left(\frac{x}{10}\right) = -0.671$
- 3 Write as a single logarithm:
a $\log 16 + 2 \log 3$ **b** $\log_2 16 - 2 \log_2 3$ **c** $2 + \log_4 5$
- 4 Write the following equations without logarithms:
a $\log T = 2 \log x - \log y$ **b** $\log_2 K = \log_2 n + \frac{1}{2} \log_2 t$
- 5 Solve for x : **a** $3^x = 300$ **b** $30 \times 5^{1-x} = 0.15$ **c** $3^{x+2} = 2^{1-x}$
- 6 If $A = \log_2 2$ and $B = \log_2 3$, write the following in terms of A and B :
a $\log_2 36$ **b** $\log_2 54$ **c** $\log_2(8\sqrt{3})$ **d** $\log_2(20.25)$ **e** $\log_2(0.\dot{8})$
- 7 For the function $g: x \mapsto \log_3(x+2) - 2$:
a Find the domain and range.
b Find any asymptotes and axes intercepts for the graph of the function.
c Sketch the graph of $y = g(x)$.
d Find g^{-1} . Explain how to verify your answer for g^{-1} .
e Sketch the graphs of g , g^{-1} and $y = x$ on the same axes.
- 8 Solve exactly for a , the equation $\log_4 a^5 + \log_2 a^{\frac{3}{2}} = \log_8 625$.

- 8 a $x+z$ b $z+2y$ c $x+z-y$ d $2x+\frac{1}{2}y$
 e $3y-\frac{1}{2}z$ f $2z+\frac{1}{2}y-3x$
 9 a 0.86 b 2.15 c 1.075
 10 a $x=9$ b $x=2$ or 4 c $x=25\sqrt{5}$ d $x=200$
 e $x=5$ f $x=3$

EXERCISE 4D

- 1 a $x \div 3.32$ b $x \div 2.73$ c $x \div 3.32$ d $x \div 37.9$
 e $x \div -3.64$ f $x \div -7.55$ g $x \div 7.64$
 h $x \div 32.0$ i $x \div 1150$
 2 a $t \div 6.340$ b $t \div 74.86$ c $t \div 8.384$
 d $t \div 132.9$ e $t \div 121.5$ f $t \div 347.4$

EXERCISE 4E

- 1 a 3.90 h b 15.5 h 2 a 66.7 min b 221 min
 3 a 25 years b 141 years c 166 years
 4 a 10 000 years b 49 800 years 5 15.8°C
 6 166 seconds 7 11.6 seconds

EXERCISE 4F

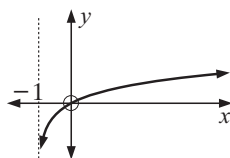
- 1 6.17 years, i.e., 6 years 62 days
 2 8.65 years, i.e., 8 years 237 days
 3 a $\frac{8.4\%}{12} = 0.7\% = 0.007$ $r = 1 + 0.007 = 1.007$
 b after 74 months

EXERCISE 4G

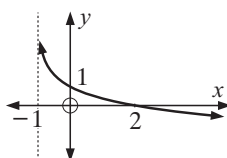
- 1 a $\div 2.26$ b $\div -10.3$ c $\div -2.46$ d $\div 5.42$
 2 a $x \div -4.29$ b $x \div 3.87$ c $x \div 0.139$
 3 a $x \div 0.683$ b $x \div -1.89$
 4 a $x = 16$ b $x \div 1.71$ 5 $x = \frac{\log 8}{\log 25}$ or $\log_{25} 8$

EXERCISE 4H

- 1 a i $x \in]-1, \infty [$, iii
 $y \in \mathcal{R}$
 ii VA is $x = -1$,
 x and y -intercepts 0
 iv $x = -\frac{2}{3}$
 v $f^{-1}(x) = 3^x - 1$

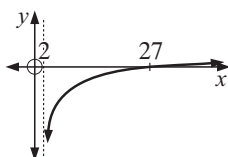


- b i $x \in]-1, \infty [$, iii
 $y \in \mathcal{R}$
 ii VA is $x = -1$,
 x -intercept 2,
 y -intercept 1

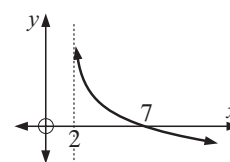


- iv $x = 8$
 v $f^{-1}(x) = 3^{1-x} - 1$

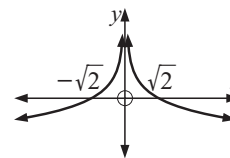
- c i $x \in]2, \infty [$, iii
 $y \in \mathcal{R}$
 ii VA is $x = 2$,
 x -intercept 27,
 no y -intercept
 iv $x = 7$
 v $f^{-1}(x) = 5^{2+x} + 2$



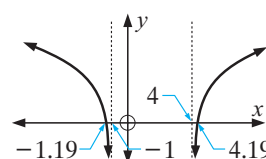
- d i $x \in]2, \infty [$, iii
 $y \in \mathcal{R}$
 ii VA is $x = 2$,
 x -intercept 7,
 no y -intercept
 iv $x = 27$
 v $f^{-1}(x) = 5^{1-x} + 2$



- e i $x \in \mathcal{R}, x \neq 0$, iii
 $y \in \mathcal{R}$
 ii VA is $x = 0$,
 x -intercepts $\pm\sqrt{2}$,
 no y -intercept
 iv $x = \pm 2$
 v if $x > 0$, $f^{-1}(x) = 2^{\frac{1-x}{2}}$
 if $x < 0$, $f^{-1}(x) = -2^{\frac{1-x}{2}}$



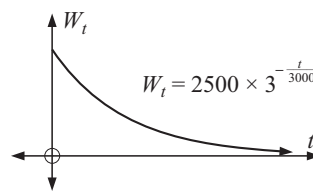
- f i $x \in]-\infty, -1 [$ iii
 or $x \in]4, \infty [$
 $y \in \mathcal{R}$
 ii VA $x = -1, x = 4$
 x -ints. 4.19, -1.19,
 no y -intercept



- iv $x = -1.10$ and 4.10
 v if $f(x) = \log_2(x^2 - 3x - 4), x > 4$,
 $f^{-1}(x) = \frac{3 + \sqrt{25 + 2^{x+2}}}{2}$
 if $f(x) = \log_2(x^2 - 3x - 4), x < -1$,
 $f^{-1}(x) = \frac{3 - \sqrt{25 + 2^{x+2}}}{2}$

REVIEW SET 4A

- 1 a 3 b 8 c -2 d $\frac{1}{2}$ e 0 f 1 g $\frac{1}{4}$ h -1
 i $\frac{1}{3}$ j $\frac{1}{2}$ 2 a $\frac{1}{2}$ b $-\frac{1}{3}$ c $a+b+1$
 3 a $x = \frac{1}{8}$ b $x \div 82.7$ c $x \div 0.0316$
 4 a $\log P = \log 3 + x \log b$ b $\log m = 3 \log n - 2 \log p$
 5 a $k \div 3.25 \times 2^x$ b $Q = P^3 R$ c $A = \frac{B^5}{400}$
 6 a $x \div 1.209$ b $x \div 1.822$
 7 a 2500 g d $W_t = 2500 \times 3^{-\frac{t}{3000}}$
 b 3290 years
 c 42.3%



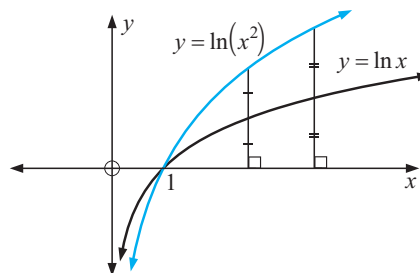
- 8 $x \div 2.32$

REVIEW SET 4B

- 1 a $\frac{3}{2}$ b $\frac{2}{3}$ c $a+b$
 2 a $x = 1000$ b $x \div 4.70$ c $x \div 6.28$
 3 a $\log 144$ b $\log_2(\frac{16}{9})$ c $\log_4 80$
 4 a $T = \frac{x^2}{y}$ b $K = n\sqrt{t}$
 5 a $x \div 5.19$ b $x \div 4.29$ c $x \div -0.839$
 6 a $2A+2B$ b $A+3B$ c $3A+\frac{1}{2}B$ d $4B-2A$
 e $3A-2B$

- 4 Kelly said that in order to graph $y = \ln(x^2)$, first graph $y = \ln x$ and double the distances away from the x -axis. Connecting these points will give the graph of $y = \ln x^2$.

Is she correct? Give evidence.



REVIEW SET 5A

- Find, to 3 significant figures, the value of:
 - e^4
 - $3e^2$
 - $\frac{1}{6e}$
 - $\frac{10}{\sqrt{e}}$
- On the same set of axes sketch and clearly label graphs of $f: x \mapsto e^x$, $g: x \mapsto e^{-x}$ and $h: x \mapsto -e^{-x}$.
 - What is the geometric connection between:
 - f and g
 - g and h ?
- Sketch on the same set of axes the graphs of $y = e^x$ and $y = 3e^x$.
- A particle moves in a straight line such that its displacement from the origin O is given by $s(t) = 120t - 40e^{-\frac{t}{5}}$ metres, where t is the time in seconds, $t \geq 0$.
 - Find the position of the particle at
 - $t = 0$
 - $t = 5$
 - $t = 20$.
 - Hence sketch the graph of $s(t) = 120t - 40e^{-\frac{t}{5}}$ for $t \geq 0$.
- Without using a calculator, find:
 - $\ln(e^5)$
 - $\ln(\sqrt{e})$
 - $\ln\left(\frac{1}{e}\right)$
- Simplify:
 - $\ln(e^{2x})$
 - $\ln(e^2 e^x)$
 - $\ln\left(\frac{e}{e^x}\right)$
- Solve for x , giving your answers to 3 significant figures:
 - $\ln x = 5$
 - $3 \ln x + 2 = 0$
- Write as a single logarithm:
 - $\ln 6 + \ln 4$
 - $\ln 60 - \ln 20$
 - $\ln 4 + \ln 1$
 - $\ln 200 - \ln 8 + \ln 5$
- Write in the form $a \ln k$ where a and k are positive whole numbers and k is prime:
 - $\ln 32$
 - $\ln 125$
 - $\ln 729$
- Solve for x , giving answers correct to 3 significant figures:
 - $e^x = 400$
 - $e^{2x+1} = 11$
 - $25e^{\frac{x}{2}} = 750$
 - $e^{2x} = 7e^x - 12$

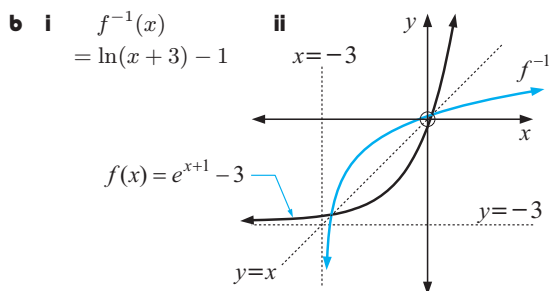
REVIEW SET 5B

Click on the icon to obtain printable review sets and answers

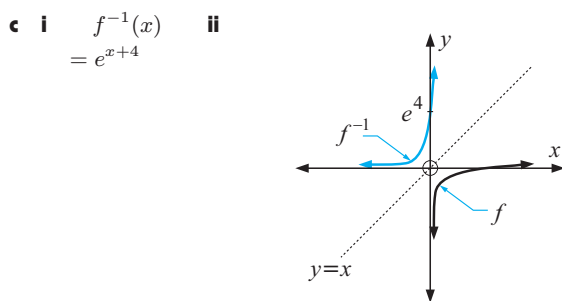
REVIEW SET 5B



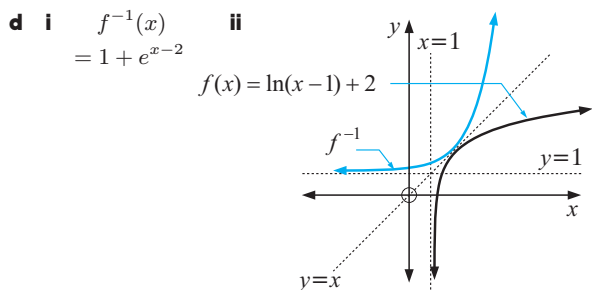
iii domain of f is $\{x : x \in \mathcal{R}\}$, range is $\{y : y > 5\}$
 domain of f^{-1} is $\{x : x > 5\}$, range is $\{y : y \in \mathcal{R}\}$



iii domain of f is $\{x : x \in \mathcal{R}\}$, range is $\{y : y > -3\}$
 domain of f^{-1} is $\{x : x > -3\}$, range is $\{y : y \in \mathcal{R}\}$

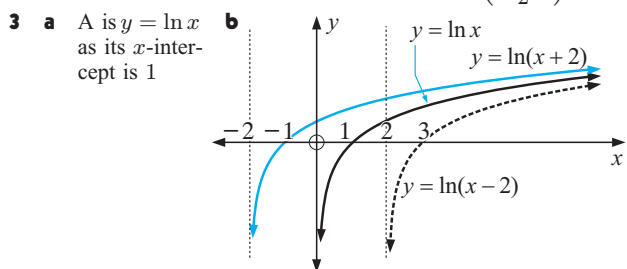


iii domain of f is $\{x : x > 0\}$, range of f is $\{y : y \in \mathcal{R}\}$
 domain of f^{-1} is $\{x : x \in \mathcal{R}\}$, range is $\{y : y > 0\}$



iii domain of f is $\{x : x > 1\}$, range is $\{y : y \in \mathcal{R}\}$
 domain of f^{-1} is $\{x : x \in \mathcal{R}\}$, range is $\{y : y > 1\}$

2 $f^{-1}(x) = \frac{1}{2} \ln x$ a $\frac{1}{2} \ln(2x-1)$ b $\frac{1}{2} \ln\left(\frac{x+1}{2}\right)$



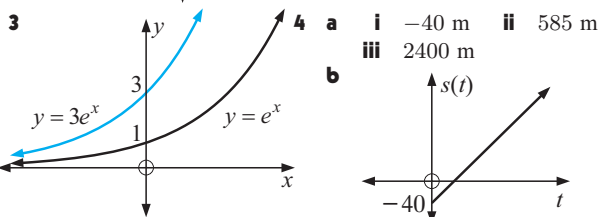
c $y = \ln x$ has VA $x = 0$
 $y = \ln(x-2)$ has VA $x = 2$
 $y = \ln(x+2)$ has VA $x = -2$

4 $y = \ln(x^2) = 2 \ln x$, so she is correct.
 This is because the y -values are twice as large for $y = \ln(x^2)$ as they are for $y = \ln x$.

REVIEW SET 5A

1 a $\div 54.6$ b $\div 22.2$ c $\div 0.0613$ d $\div 6.07$

2 a b i g is the reflection of f in the y -axis
 ii h is the reflection of g in the x -axis

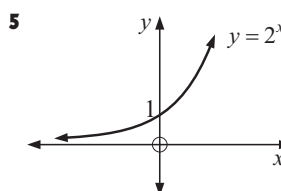
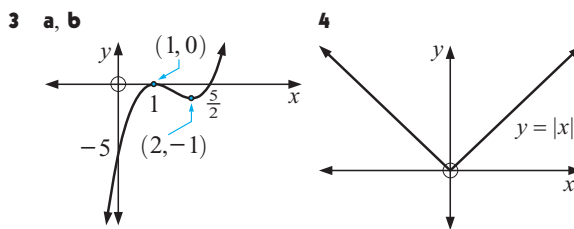
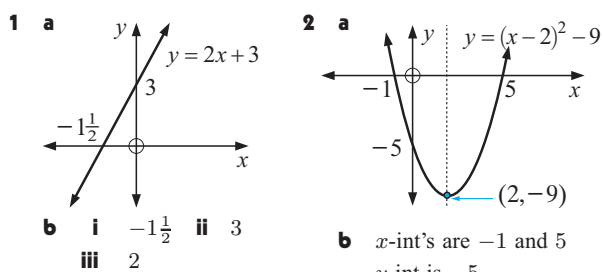


5 a 5 b $\frac{1}{2}$ c -1 6 a $2x$ b $2+x$ c $1-x$
 7 a $x \div 148$ b $x \div 0.513$
 8 a $\ln 24$ b $\ln 3$ c $\ln 4$ d $\ln 125$
 9 a $5 \ln 2$ b $3 \ln 5$ c $6 \ln 3$
 10 a $x \div 5.99$ b $x \div 0.699$ c $x \div 6.80$
 d $x \div 1.10$ or 1.39

EXERCISE 6A

1 a $2x$ b $x+2$ c $\frac{x}{2}$ d $2x+3$
 2 a $64x^3$ b $4x^3$ c x^3+3x^2+3x+1
 d $2x^3+6x^2+6x-1$
 3 a 4^x b $2^{-x}+1$ c $2^{x-2}+3$ d $2^{x+1}+3$
 4 a $-\frac{1}{x}$ b $\frac{2}{x}$ c $\frac{2+3x}{x}$ d $\frac{2x+1}{x-1}$

EXERCISE 6B



When $x = 0$,
 $y = 2^0 = 1$ ✓
 $2^x > 0$ for all x as
 the graph is always
 above the y -axis. ✓

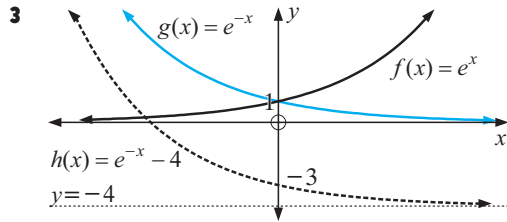
REVIEW SET 5B

- 1** Write as a single power of e :
- a** $\frac{1}{e^3}$ **b** $\frac{\sqrt{e}}{e^2}$ **c** $e^3\sqrt{e^3}$ **d** $\sqrt{10e}$
- 2** Evaluate correct to 6 significant figures:
- a** $4.8e^{1.725}$ **b** $0.66e^{-1.7968}$
- 3** On the same set of axes, sketch and clearly label the graphs of
 $f: x \mapsto e^x$, $g: x \mapsto e^{-x}$, $h: x \mapsto e^{-x} - 4$.
 State the domain and range of each function.
- 4** Sketch on the same set of axes, the graphs of $y = e^x$ and $y = e^{3x}$.
- 5** Without using a calculator, find:
- a** $\ln(e\sqrt{e})$ **b** $\ln\left(\frac{1}{e^3}\right)$ **c** $\ln\left(\frac{e}{\sqrt{e^5}}\right)$
- 6** Write in the form e^x where x is correct to 3 significant figures:
- a** 20 **b** 3000 **c** 0.075
- 7** Simplify:
- a** $4\ln 2 + 2\ln 3$ **b** $\frac{1}{2}\ln 9 - \ln 2$ **c** $2\ln 5 - 1$ **d** $\frac{1}{4}\ln 81$
- 8** Write the following equations without logarithms:
- a** $\ln P = 1.5\ln Q + \ln T$ **b** $\ln M = 1.2 - 0.5\ln N$
- 9** Consider $g: x \mapsto 2e^x - 5$.
- a** Find the defining equation of g^{-1} .
b Sketch the graphs of g and g^{-1} on the same set of axes.
c State the domain and range of g and g^{-1} .
- 10** The weight W_t grams of radioactive substance remaining after t weeks is given by
 $W_t = 8000 \times e^{-\frac{t}{20}}$ grams. Find the time for the weight to:
- a** halve **b** reach 1000 g **c** reach 0.1% of its original value.

REVIEW SET 5B

1 a e^{-3} b $e^{-1.5}$ c $e^{4.5}$ d $\doteq e^{1.65}$

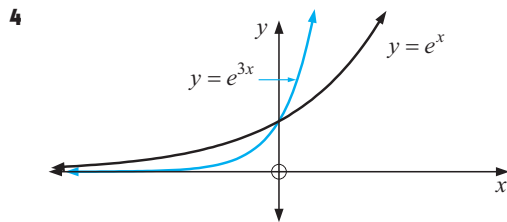
2 a $\doteq 26.9401$ b $\doteq 0.109447$



Each function has domain $\{x : x \in \mathcal{R}\}$

Range of f is $\{y : y > 0\}$, Range of g is $\{y : y > 0\}$

Range of h is $\{y : y > -4\}$



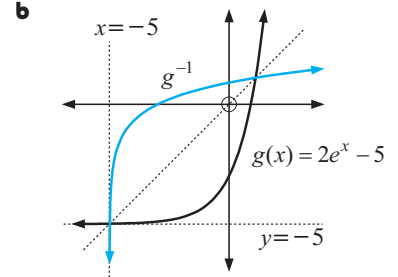
5 a $\frac{3}{2}$ b -3 c $-\frac{3}{2}$

6 a $\doteq e^{3.00}$ b $\doteq e^{8.01}$ c $\doteq e^{-2.59}$

7 a $\ln 144$ b $\ln(\frac{3}{2})$ c $\ln(\frac{25}{e})$ d $\ln 3$

8 a $P = TQ^{1.5}$ b $M = \frac{e^{1.2}}{\sqrt{N}}$

9 a $g^{-1}(x)$
 $= \ln\left(\frac{x+5}{2}\right)$



c domain of g is $\{x : x \in \mathcal{R}\}$, range is $\{y : y > -5\}$

domain of g^{-1} is $\{x : x > -5\}$, range is $\{y : y \in \mathcal{R}\}$

10 a 13.9 weeks b 41.6 weeks c 138 weeks